CBCS SCHEME

USN

20EVE/ESP/EIE/ELD/ECS11

First Semester M.Tech. Degree Examination, Feb./Mar. 2022
Advanced Engineering Mathematics

Time: 3 hrs. Max. Marks: 100

Note: Answer any FIVE full questions, choosing ONE full question from each module.

Module-1

1 a. Define: (i) Vector space (ii) Subspace. Show that the set $w = \{(x, y, z)/x - 3y + 4z = 0\}$ is a subspace of the vector space $V_3(R)$. **(07 Marks)**

b. Define a basis of vector space. Show that the set $B = \{(1, 1, 0), (1, 0, 1), (0, 1, 1)\}$ is a basis for the vector space $V_3(R)$. **(07 Marks)**

c. If $T:V_1(R) \to V_3(R)$ is defined by $T(X) = (x, x^2, x^3)$. Find whether T is a linear transformation or not. **(06 Marks)**

OR

2 a. Define a linear transformation. A transformation $T: R^2 \to R^3$ is defined by $T(X) = AX$ so that

$$T(X) = AX = \begin{bmatrix} 1 & -3 \\ 3 & 5 \\ -1 & 7 \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} x_1 - 3x_2 \\ 3x_1 + 5x_2 \\ -x_1 + 7x_2 \end{bmatrix} \text{ and } b = \begin{bmatrix} 3 \\ 2 \\ -5 \end{bmatrix} \quad c = \begin{bmatrix} 3 \\ 2 \\ 5 \end{bmatrix}.$$

Find:

(i) The image T(u), of u under the transformation where $u = \begin{bmatrix} 2 \\ -1 \end{bmatrix}$.

(ii) The X is R^2 whose image under T is b. Is this X unique under T having image b?

(iii) Determine whether C is in the range of transformation T. **(07 Marks)**

b. Find a basis for R^4 that contains the vectors $V_1 = (1, 0, 1, 0)$ and $V_2 = (-1, 1, -1, 0)$. **(07 Marks)**

c. If $T : R^2 \to R^3$ is defined by $T(x, y) = (x + y, x, 3x - y)$ with respect to the basis $B_1 = \{(1, 0), (0, 1)\}$ $B_2 = \{(1, 0, 0), (0, 1, 0), (0, 0, 1)\}$. Find the matrix of linear transformation. **(06 Marks)**

Module-2

3 a. Find the eigen values and the corresponding eigen vectors by using Given's method for the matrix, $A = \begin{bmatrix} 1 & 2 & 3 \\ 2 & 1 & -1 \\ 3 & -1 & 1 \end{bmatrix}$. **(10 Marks)**

b. Apply Gram-Schmidt orthogonalization process to find an orthonormal basis for the subspace R^4 spanned by the vectors $(1, 1, 1, 0), (-1, 0, -1, 1), (-1, 0, 0, -1)$. **(10 Marks)**

OR

4 a. By using Given's method, find the eigen values and the corresponding eigen vectors to the matrix $A = \begin{bmatrix} 2 & 3 & 1 \\ 3 & 2 & 2 \\ 1 & 2 & 1 \end{bmatrix}$. **(10 Marks)**

b. Define: (i) Orthogonal vectors (ii) Orthonormal basis. Using Gram-Schmidt process, construct an orthogonal set of vectors from linearly independent set $\{X_1, X_2, X_3\}$ where

$$X_1 = \begin{pmatrix} -4 \\ 3 \\ 6 \end{pmatrix} \qquad X_2 = \begin{pmatrix} 2 \\ -3 \\ 6 \end{pmatrix} \qquad X_3 = \begin{pmatrix} 2 \\ 3 \\ 0 \end{pmatrix}$$

(10 Marks)

Module-3

5 a. Define a functional. Derive Euler's equation in the form $\frac{\partial f}{\partial y} - \frac{d}{dx}\left(\frac{\partial f}{\partial y'}\right) = 0$. **(07 Marks)**

b. Find the shortest distance between the point A(1, 0) and the Ellipse $4x^2 + 9y^2 = 36$. **(07 Marks)**

c. Find the extremal of the function I under the conditions $y(0) = 0$, $y\left(\frac{\pi}{2}\right) = 0$ where

$$I = \int_0^{\pi/2} (y^2 - (y')^2 - 2y\sin x)dx.$$

(06 Marks)

OR

6 a. A heavy cable hangs freely under gravity between two fixed points. Show that the shape of the cable is a catenary. **(07 Marks)**

b. Show that the curve which extremizes the functional $I = \int_0^{\pi/4} ((y'')^2 - y^2 + x^2)dx$ under the conditions $y(0) = 0$, $y'(0) = 1$, $y\left(\frac{\pi}{4}\right) = \frac{1}{\sqrt{2}}$, $y'\left(\frac{\pi}{4}\right) = \frac{1}{\sqrt{2}}$ is $y = \sin x$. **(07 Marks)**

c. Prove that the sphere is the solid figure of revolution for a given surface area, has maximum volume. **(06 Marks)**

Module-4

7 a. Find the characteristic function of the Poisson distribution. Also find the first four central moments. **(07 Marks)**

b. In a certain city the daily consumption of electric power in millions of kilowatt-hrs can be treated as a random variable having an Erlong distribution with parameters $\lambda = \frac{1}{2}$, $K = 3$. If the power plant of this city has a daily capacity of 12 million kilo-watt-hrs, what is the probability that this power supply will be inadequate on any given day? **(07 Marks)**

c. In a normal distribution 31% items are under 45, 8% are over 64. Find the mean and the standard deviation, given that $\phi(0.5) = 0.19$, $\phi(1.4) = 0.42$. **(06 Marks)**

OR

8 a. Define:
(i) Characteristic function
(ii) Moment generating function
(iii) Probability generating function

Find the moment generating function for the function $f(x) = \frac{1}{C}e^{-x/c}$, $0 \le x \le \infty$, $C > 0$.

(07 Marks)

b. A random variable X has the probability distribution function :

X	−2	−1	0	1	2	3
P(X)	0.1	K	0.2	2K	0.3	K

Find the values of K. Also find mean $\overline{x}$, μ_3 and μ_4. **(07 Marks)**

c. Marks of 1000 students in an examination follows normal distribution with mean 70 and standard deviation 5. Find the number of students:
(i) Scoring less than 65 (ii) More than 75 (iii) Between 65 and 75
Given $\phi(1) = 0.3413$ or $A(1) = 0.3413$. **(06 Marks)**

Module-5

9 a. In the fair coin experiment, $\{X(t)\} = \begin{cases} \sin \pi t, & \text{if head shows} \\ 2t, & \text{if tail shows} \end{cases}$. Find $E[X(t)]$ and $F(x, t)$ for $t = 0.25$. **(07 Marks)**

b. The process $\{X(t)\}$ is normal with $\mu_t = 0$ and $R_X(\tau) = 4e^{-3|\tau|}$. Find a memoryless system $g(x)$ such that the first order density $f_y(y)$ of the resulting output $y(t) = g[X(t)]$ is uniform in the interval (6, 9). **(07 Marks)**

c. Define:
(i) First order stationary process
(ii) Second order stationary process
(iii) Wide-sense stationary process **(06 Marks)**

OR

10 a. Show that the process $X(t) = A\cos\lambda t + B\sin\lambda t$ is wide-sense stationary, if
(i) $E(A) = E(B) = 0$ (ii) $E(A^2) = E(B^2)$ **(07 Marks)**

b. Given that the auto-correlation function for a stationary ergodic process with no periodic components is $R_{XX}(\tau) = 25 + \dfrac{4}{1 + 6\tau^2}$. Find the mean value and variance of the process [X(t)]. **(07 Marks)**

c. Show that the random process $X(t) = A\cos(\omega_0 t + \theta)$ is wide-sense stationary if A and ω_0 are constants and θ is a uniformly distributed random variable in $(0, 2\pi)$. **(06 Marks)**

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